Next-generation signal processing using windowed Nonlinear Fourier Transform

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ABSTRACT

This article introduces an innovative procedure integrating chromatic dispersion compensation (CDC) and sliding window methodology for ongoing signal processing in optical communications. Our strategy notably elevates the effectiveness of Nonlinear Fourier Transform (NFT) processing.

Keywords: Nonlinear Fourier Transform, Optical Communication, Digital Backpropagation, Chromatic Dispersion Compensation, Nonlinear Equalisation

1. INTRODUCTION

The Nonlinear Fourier Transform (NFT) is a mathematical instrument employed to process signals in fiber-optic networks, benefiting from its unique three-step approach.[1] This process enables intricate modeling of chromatic dispersion and nonlinear effects such as the Kerr effect, which are essential in long-distance signal propagation.

NFT's capabilities improve the efficiency of optical communication systems and increase transmission capacity by offering a detailed understanding of signal distortion.[2–5] However, its application is not without challenges, primarily due to its high computational demands and the necessity for specialized hardware or software, thereby increasing system complexity and costs.

Regardless, the potential benefits of NFT in optical communications mark it as an exciting research area, potentially revolutionizing future signal processing and transmission.

Handling continuous signals with NFT, though, remains challenging. Traditional sliding window techniques for signal division prove insufficient. To combat this, we suggest an innovative approach amalgamating chromatic dispersion compensation (CDC) and sliding window techniques for continuous signal processing. By compensating chromatic dispersion beforehand, we mitigate overlapping and distortion between symbols, thereby enhancing NFT performance.[6–8] This method increases accuracy, addresses continuous signal processing challenges, and ultimately improves overall system performance and transmission capacity in optical telecommunications.

2. SLIDING WINDOW AND COMPENSATED WINDOW PROCESSING

Considering a continuous signal, $A(z = L, t) = A(t)$, desired to be processed via NFT, the signal's dispersion must be accounted for to ensure accurate processing. Dispersion leads to an overlap and distortion of neighboring symbols, affecting the signal processing accuracy. To overcome this, side dispersion intervals T_d are incorporated to both sides of the processing interval T_{proc}. The dispersion interval T_d depends on the propagation distance and is estimated as $T_d = \beta_2 \times \Omega \times L$.

Figure 1 depicts the schematic representation of each processing stage in the approach. The signal is partitioned using the window function $W(t)$, defined as:

$$W(t, t_w) = \begin{cases} 1, & t_w - T_d \le t \le t_w + T_{proc} + T_d, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

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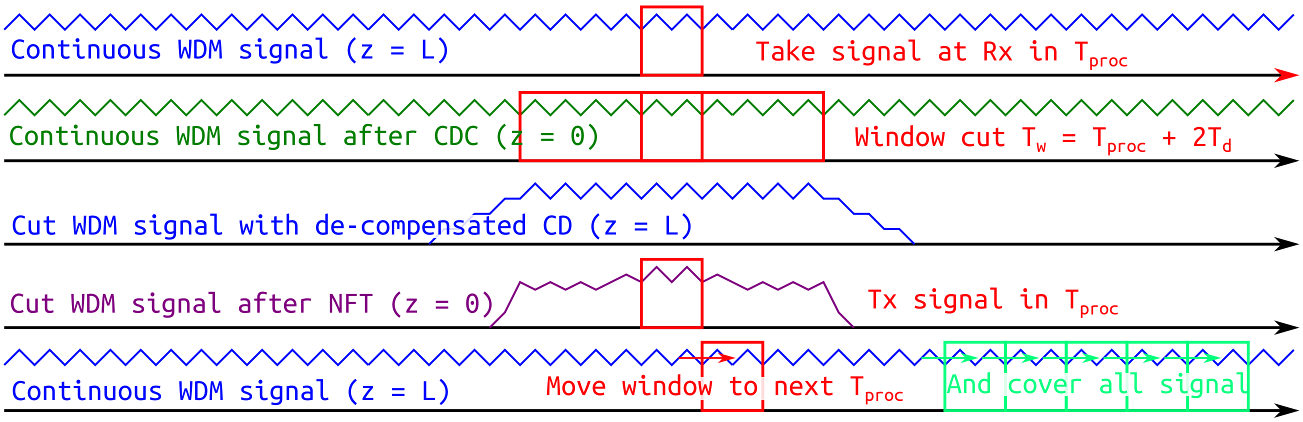

Figure 1. Schematic representation of the processing procedure for a continuous WDM signal utilizing NFT. The initial step involves performing CDC for the entire signal. Next step is extraction of a window of size T_w and compensation for the dispersion. The resulting signal undergoes NFT processing to recover the transmitted signal. This procedure is repeated iteratively.

The signal $A_w(t)$ inside the $T_d + T_{proc} + T_d$ window is then given by $A_w(t, t_w) = A(t)W(t, t_w)$, which undergoes NFT processing to restore the initial transmitted signal. However, the side intervals T_d may be corrupted as the signal information from previous and following intervals is lost during windowing.

To ensure the processed interval retains all information about nonlinear distortions and other side interval effects, the T_{proc} interval is recut, yielding:

$$W_{proc}(t, t_w) = \begin{cases} 1, & t_w \leq t \leq t_w + T_{proc}, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

This processed signal is obtained as $A_w(z = 0, t, t_w) = W_{proc}(t, t_w) \cdot \mathrm{NFT}[A_w(z = L, t, t_w)]$. By advancing t_w by the value of the processing interval T_{proc}, the procedure can be repeated for the next window position. Although effective, numerical accuracy may sometimes be suboptimal, requiring improvements like zero intervals around the processing signal.

To increase the stability of numerical calculations, we propose a compensated window method, leveraging the signal's propagation model to "compress" the signal before processing by compensating for chromatic dispersion. After dispersion compensation, the signal is windowed and dispersion decompensated to yield a signal ready for NFT analysis.

First, the entire signal's dispersion is compensated as $A_{CD}(t) = F^{-1}[F[A(t)]e^{i\phi_{CD}(\omega)}](t)$, where $\phi_{CD} = -\frac{\beta_2}{2}\omega^2 L$. After dispersion compensation, the same window operation is applied, and dispersion is decompensated using the same phase shift to obtain $A_{w,CD}(t, t_w) = A_{CD}(t)W(t, t_w)$ and $A_{\tilde{w}}(t) = F^{-1}[F[A_{w,CD}]e^{-i\phi_{CD}(\omega)}](t)$.

The final solution for the processing window using the dispersion-compensated window mode is $A_{\tilde{w}}(z = 0, t, t_w) = W_{proc}(t, t_w) \cdot \mathrm{NFT}[A_{\tilde{w}}(z = L, t, t_w)]$. By incrementing $t_w = t_w + T_{proc}$, the entire signal can be processed continuously.

3. RESULTS

We evaluated the performance of our proposed method by simulating a single channel wavelength division multiplexing (WDM) signal in an optical communication system.[9] The system consisted of 16-QAM symbols with single polarization and a symbol rate of 34.4 [GBd], shaped by a root raised cosine (RRC) filter with a roll-off factor of $\rho = 0.1$.

Simulations involved transmitting the signal over 12×80 [km] spans of standard single-mode fiber (SSFM), complemented with distributed Raman amplification (DRA). Two scenarios were considered: one noiseless scenario and another with DRA noise, mimicking an erbium-doped fiber amplifier noise figure of 4.5 [dB]. The fiber had an attenuation coefficient of $\alpha = 0$ [dB/km], a dispersion coefficient of $D = 16.8$ [ps/{nm · km}], and a nonlinear coefficient of $\gamma = 1.2$ $[\mathrm{W} \cdot \mathrm{km}]^{-1}$. Calculations were performed with 2 samples per symbol.

The results were analyzed in terms of Q-factor improvement and the level of HD-FEC (Hard Decision Forward Error Correction), which represents a common error correction scheme used in optical communication systems. The target Bit Error Rate (BER) was set at 4%. We assessed the Q-factor for conventional digital signal processing (DSP) techniques such as digital backpropagation (DBP) and chromatic dispersion compensation (CDC) against the NFT-based window processing method. DBP2, DBP3, and DBP10 represent DBP with 2, 3, and 10 steps per span, respectively. Fig. 2 illustrates the performance of NFT.

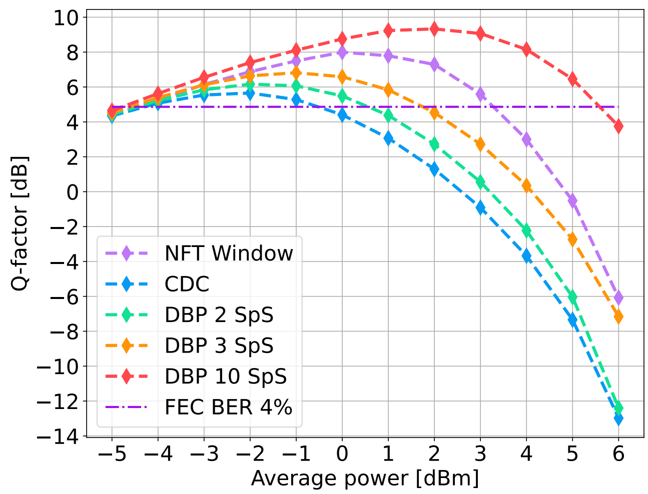

Figure 2. Dependence of Q-factor on the average signal power P_0. The different lines correspond to different processing methods: NFT Window (proposed approach), CDC (Chromatic Dispersion Compensation with nonlinear phase equalisation), DBP2, DBP3, DBP10 (Digital Backpropagation with 2, 3, and 10 steps per span respectively), and FEC (Forward Error Correction) for BER 4%.

Without noise, DBP methods can struggle at high average signal power. In most scenarios of average input signal power, the NFT with window processing (NFTWindow) surpasses DBP3, achieving a higher Q-factor. This result underscores the efficiency of NFTWindow in handling nonlinear distortions while maintaining the accuracy of continuous signal processing. Moreover, the success of the NFTWindow technique showcases its potential for addressing the challenges inherent in optical telecommunications and enhancing the overall performance of communication systems.

4. CONCLUSION

The Nonlinear Fourier Transform (NFT) offers a compelling framework for analyzing and processing signals in fiber-optic networks. However, its application to continuous signals introduces considerable challenges. In response to these, we have proposed an innovative approach combining chromatic dispersion compensation (CDC) and a sliding window technique. This fusion significantly elevates the precision and efficiency in the handling of continuous signals.

Our simulation results indicate that the proposed NFT sliding window methodology surpasses traditional digital signal processing strategies, such as digital backpropagation, under diverse conditions of average input signal power. These findings underscore the potential of our method to enhance the overall performance of optical telecommunications.

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